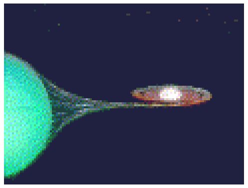
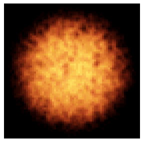




Compact Stars with Exotic States of Matter

A basic (but hopefully interesting) introduction
to matter under extreme conditions
Second Lecture

Hampton University Graduate Studies
Thomas Jefferson National Accelerator Facility
2004 June 3
Mark Paris (JLab)

Outline

- Lect 2 – The layers of the “onion” – Exotic states of matter
- EoS of nuclear matter
  - realistic potentials
  - solving the Schrodinger equation variationally
  - cold catalyzed nucleon matter
- Exotic states of matter
  - unpaired quark matter
  - CFL
  - other varieties
- Building a “realistic” star
  - equations of state
  - phase transitions in nuclear and quark matter
  - maximum mass limits

Nuclear matter

- Dense nucleon matter
  - $0.1<\rho<5$ to $10\rho_0$; $\rho_0=0.16\ \text{fm}^{-3}$
  - proton fraction, $0.01<x_p<0.15$; $x_p = \rho_p/\rho$; $\rho=\rho_n+\rho_p$
  - pure neutron matter (PNM) $x_p=0$
  - symmetric nuclear matter (SNM) has equal numbers of protons and neutrons, $x_p=0.5$
  - use many-body techniques to solve PNM, SNM and then interpolate to general x_p
- "Realistic" models of the nucleon-nucleon (NN) potential
  - at large distances NN interaction dominated by π-exchange
  - meson exchange-motivated at short distances
  - multi-meson exchanges at large distance
  - three-body two pion exchange interaction (Fujita-Miyazawa)
- Parametrize of the potential
  - fits to NN elastic scattering phase shifts up to » 350 MeV
  - fit to deuteron binding energy
  - three-body potential needed – fit to ^{3}H and equilibrium density of nuclear matter

Nuclear forces

- without nuclear forces M_{max} » 0.7 M–
- strong repulsion at small r, intermediate » 1-1.5 fm attraction

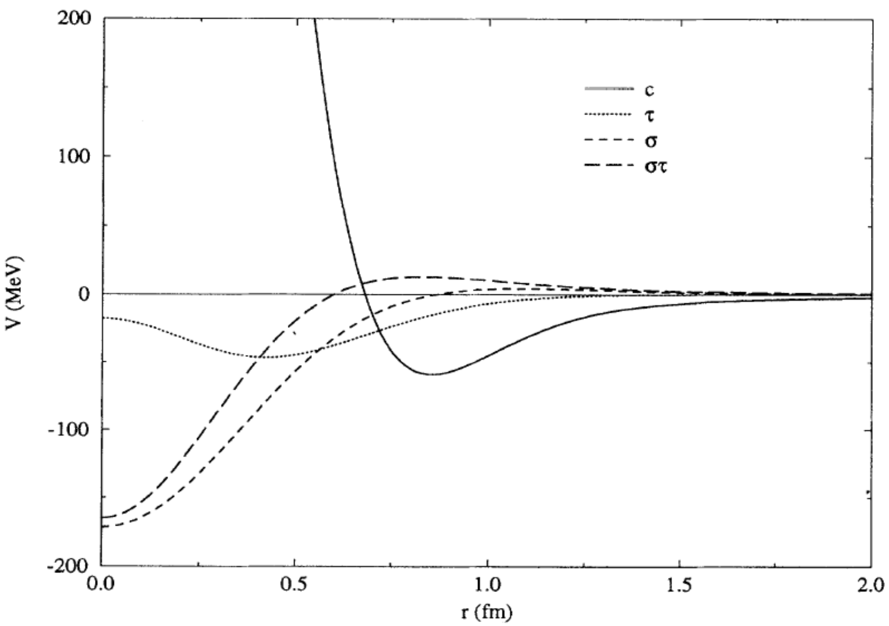


FIG. 6. Central, isospin, spin, and spin-isospin components of the potential. The central potential has a peak value of 2031 MeV at $r = 0$.

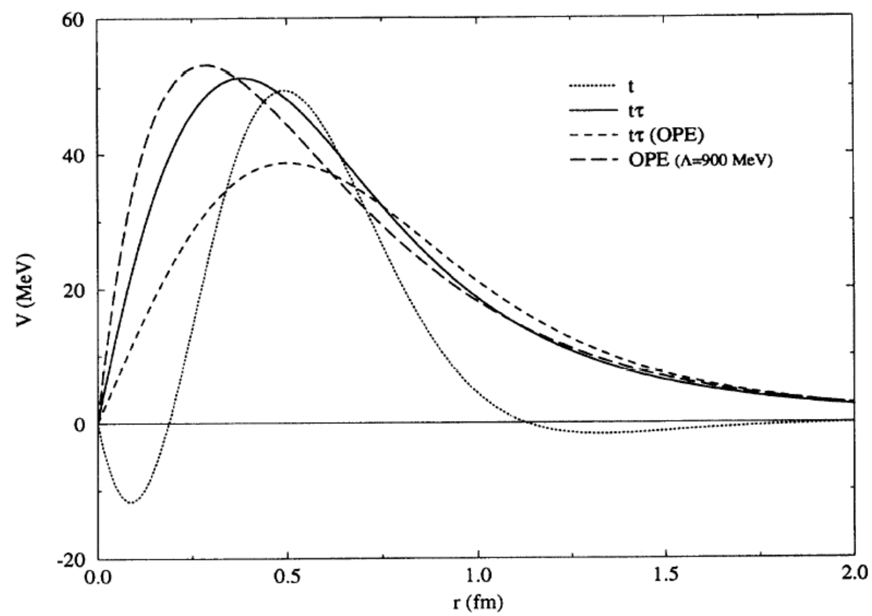


FIG. 7. Tensor and tensor-isospin parts of the potential. Also shown are the OPE contribution to the tensor-isospin potential, and for comparison an OPE potential with a monopole form factor containing a 900 MeV cutoff mass.

Realistic potentials

- NN interaction fits to scattering data with χ^2/N»1
- Egs.
  - Reid93 – local, non-relativistic
  - Paris – local, non-relativistic
  - Argonne v_{18} (Av18) – local, non-relativistic, charge dependence
  - Bonn – relativistic meson exchange w/short range cut-offs, non-local, charge dependence

Charge symmetry: n$p   Charge dependence: nn $\neq$ pp

$$
\begin{aligned}
|T=0, T_z=0\rangle &= \frac{1}{\sqrt{2}}(pn - np) \\
|T=1, T_z=-1\rangle &= nn \\
|T=1, T_z=0\rangle &= \frac{1}{\sqrt{2}}(pn + np) \\
|T=1, T_z=+1\rangle &= pp
\end{aligned}
$$

Argonne v_{14} Hamiltonian + TNI

- kinetic energy operator

$$T = \sum_i -\frac{\hbar^2}{4}\left[\left(\frac{1}{m_p}+\frac{1}{m_n}\right)+\left(\frac{1}{m_p}-\frac{1}{m_n}\right)\tau_{z,i}\right]\nabla_i^2$$

- potential operator

$$v_{ij} = \sum_{p=1}^{18} v^p(r_{ij})\mathcal{O}_{ij}^p$$

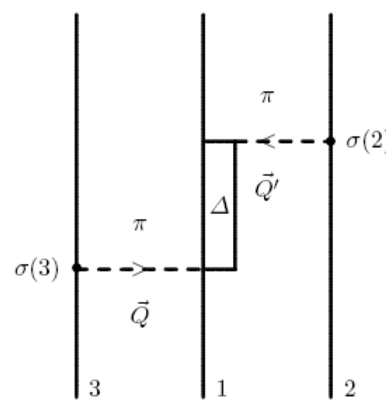
$$\mathcal{O}_{ij}^{p=1,14} = [1, \vec{\sigma}_i\cdot\vec{\sigma}_j, S_{ij}, (\vec{L}\cdot\vec{S})_{ij}, L_{ij}^2, L_{ij}^2\vec{\sigma}_i\cdot\vec{\sigma}_j, (\vec{L}\cdot\vec{S})_{ij}^2]\otimes[1, \vec{\tau}_i\cdot\vec{\tau}_j]$$

- Many-body Hamiltonian

$$H = T + \sum_{ij} v_{ij} + \sum_{ijk} V_{ijk} + \ldots$$

- Fujita-Miyazawa

$$V_{ijk}^{2\pi} =$$



Variational calculation of the energy

- obtain upper bound on ground state energy and wave function
- pair interactions induce correlations with same operator structure

$$
\begin{aligned}
|\Psi(\vec{R})\rangle &= \mathcal{S}\prod_{i<j}\widehat{F}_{ij}|\Phi\rangle \\
\widehat{F}_{ij} &= \sum_{p} f^{p}(r_{ij})\mathcal{O}^{p}_{ij}
\end{aligned}
$$

- correlation operators, F_{ij}, satisfy Euler-Lagrange equations which minimize the two-body contribution to the energy
- variational parameters, λ^p, are chosen to satisfy boundary conditions:

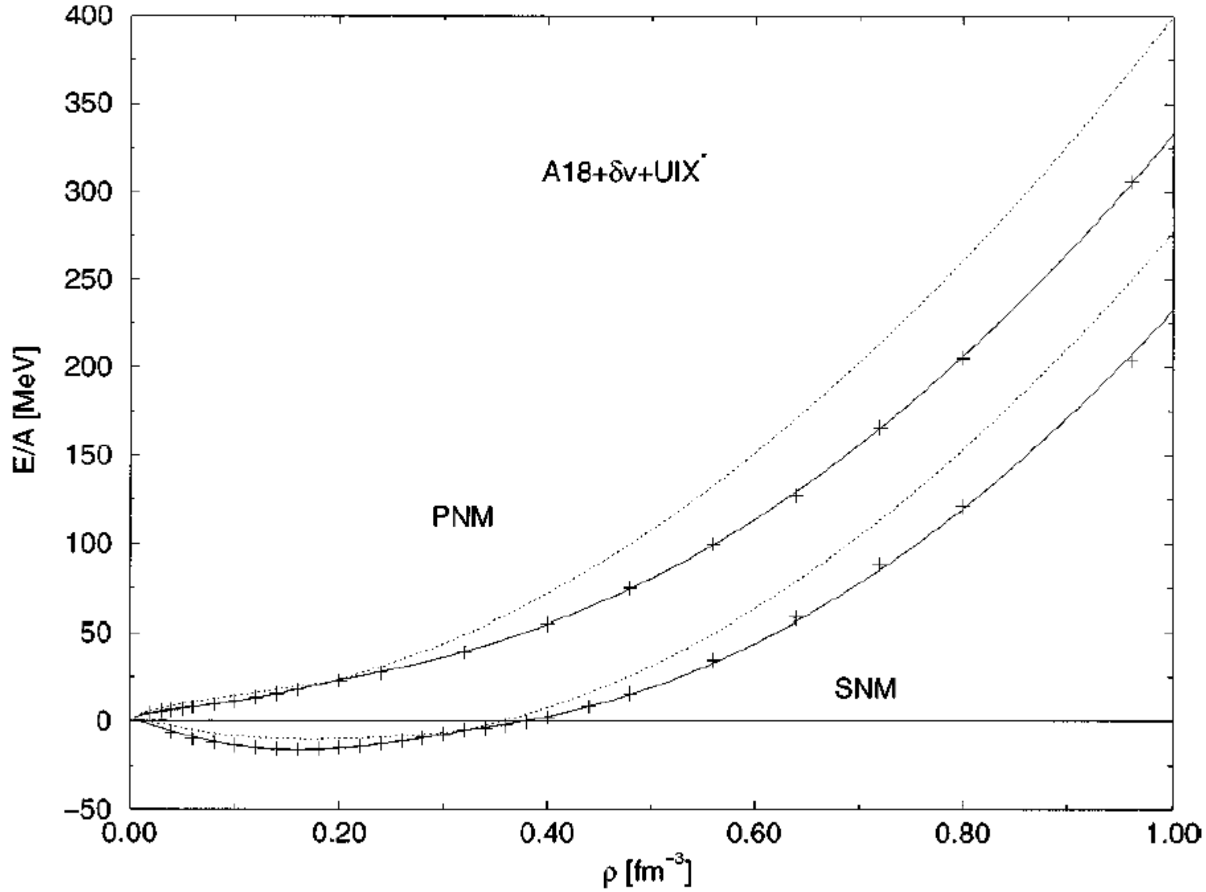
$$p = 1, \mathcal{O}^{p=1} = 1, f^{p}(r > d^{p}) = \delta_{p,1}$$

- cluster expansion of energy to two-body level: $\langle\Psi|H|\Psi\rangle$

$$f^{q}(r_{ij})\mathcal{O}^{q}_{ij}v^{p}_{ij}\mathcal{O}^{p}_{ij}f^{q'}(r_{ij})\mathcal{O}^{q'}_{ij}$$

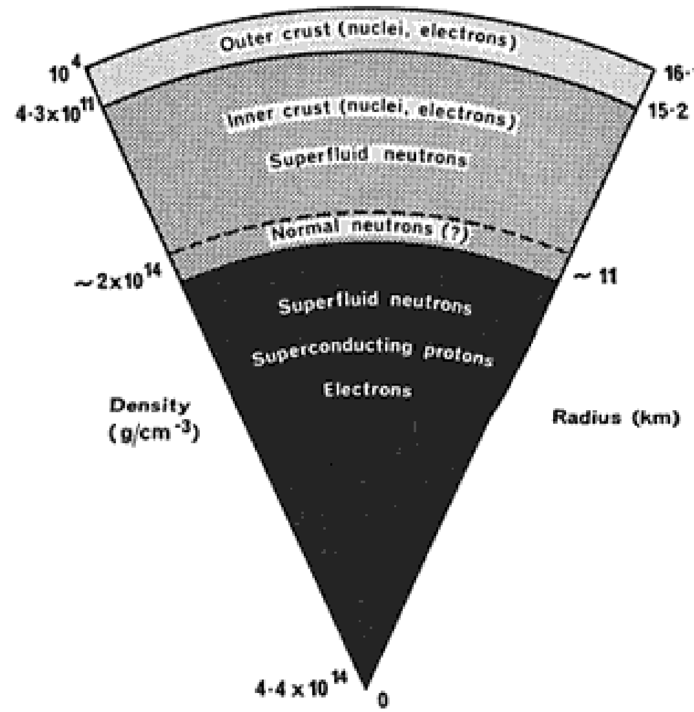
PNM & SNM energies

- energy per nucleon MeV/fm^3
- phase transition to pion condensed phase
  - PNM » 0.2 fm^{-3}
  - SNM » 0.32 fm^{-3}
- condensed phase is a high density phase
- note kink in P/SNM
- occurs when in-medium effective pion mass!0 [Migdal, RMP, 50, p107]
- important for cooling and neutron star evolution



Akaml and Pandharipande, PRC 56, p 2261
Akmal, Pandharipande, & Ravenhall, PRC 58, p 1804

Neutron star structure



- matter at T=0 in the ground state
- The Crust
  - surface, zero pressure – ^{56}Fe
  - e^-+p!n+ν as pressure increases
  - increase depth from surface) more neutron rich nuclei
  - cross neutron-drip line n»2£10^{-4}
  - as p", 30<Z<40, A" until n»0.06fm^{-3}
  - pasta phases when V_{nuclei} ¼ $V_{neutron\ gas}$
  - at n»0.1 fm^{-3} no nuclei: cold catalyzed nucleon matter
  - mass fraction in crust . 2%
- Inner crust/Outer core
  - neutron liquid with small fraction of protons
    - charge neutral – $\rho_p=\rho_e+\rho_\mu$
    - beta equilibrium – $\mu_n=\mu_p+\mu_e$, $\mu_e=\mu_\mu$
  - transition from normal phase of neutrons and protons to high density pion condensed phase over ¼ few 10’s m

Cold catalyzed nucleon matter

- Solve for equilibrium conditions

$$\begin{aligned} p &= \sum_i \mu_i n_i - \epsilon, \quad \mu_i = \frac{\partial \epsilon}{\partial \rho_i}, \quad i = p, n, e, \mu \\ p &= p_N + p_e + p_\mu \\ \epsilon &= \epsilon_N + \epsilon_e + \epsilon_\mu \end{aligned}$$

- Effective Hamiltonian for $x_p \neq 0, 0.5$

$$\begin{aligned} \epsilon_N(\rho, x_p) &= \left(\frac{\hbar^2}{2m} + f(\rho, x_p)\right)\tau_p \\ &+ \left(\frac{\hbar^2}{2m} + f(\rho, 1 - x_p)\right)\tau_n \\ &+ g(\rho, x_p = 0.5)(1 - (1 - 2x_p)^2) \\ &+ g(\rho, x_p = 0)(1 - 2x_p)^2 \end{aligned}$$

effective mass terms

symmetry energy

Cold catalyzed nucleon matter (II)

- conditions of beta equilibrium) $x_p(\rho)$

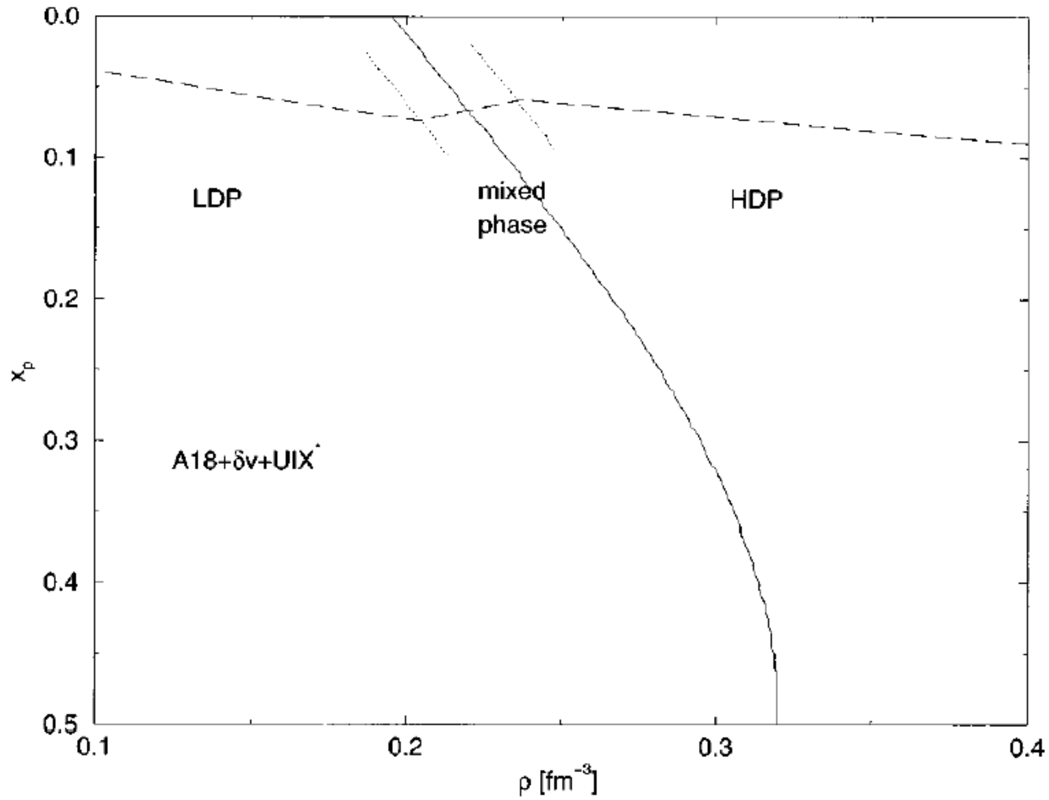
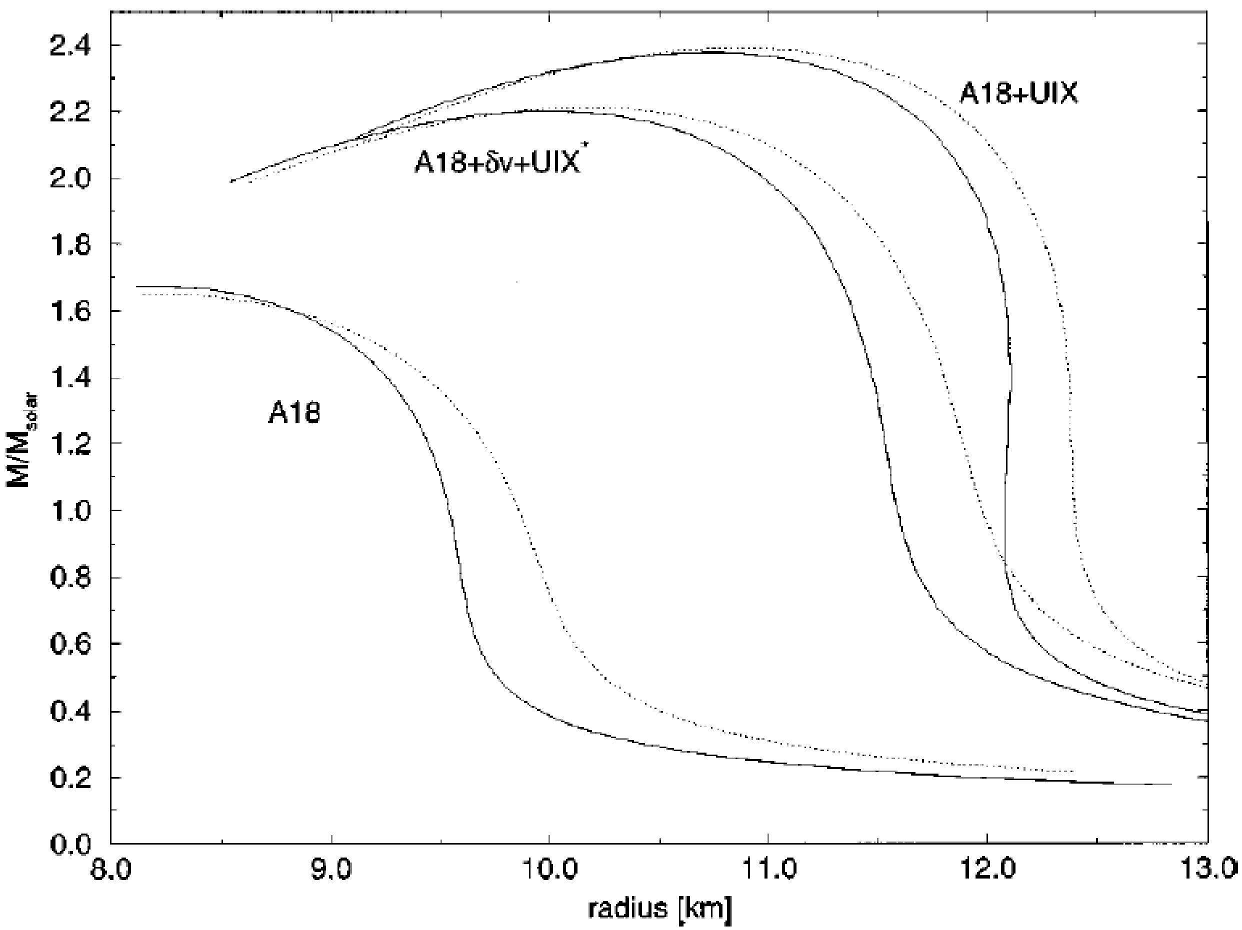


FIG. 7. On a plot of proton fraction x_p vs baryon density, for the A18+ δv +UIX* model, the boundary between the LDP and HDP, obtained in the manner described in the text. The dashed curve is the proton fraction of beta-stable matter, and the dotted lines mark the boundary of the mixed phase region.

Solve T-O-V equation

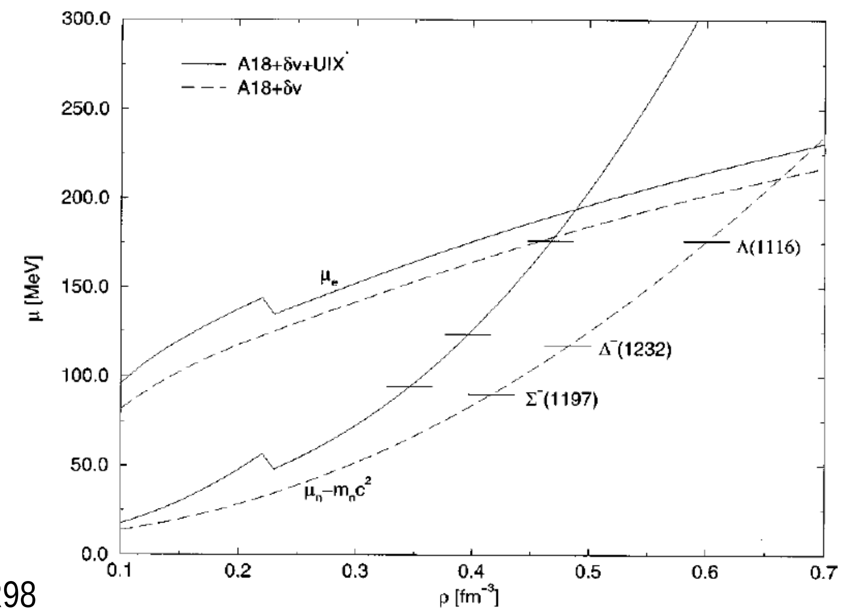
- obtain M(R)
- max mass » 2.2 M–



- effect of TNI
- effect of rel. corrections
- solid curves: beta stable
- dashed curves: PNM

Transition to quark matter

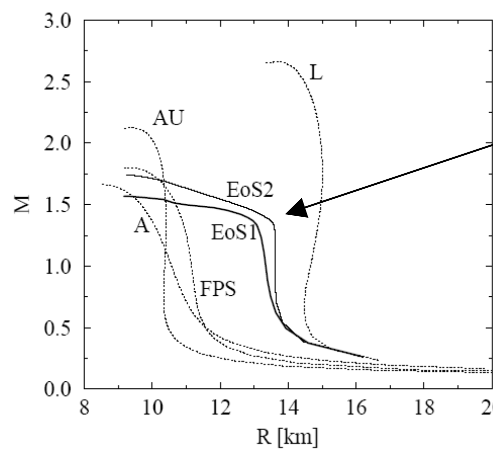
- Superluminality problematic for nuclear EoS's
  - non-relativistic
  - neglects other species
- Hyperonic matter: Λ, $\Sigma^{-,0,+}$, $\Delta^{-,0,+,++}$
  - more degrees of freedom) softer EoS
- Neglect interactions between nucleons and hyperons



APR98

FIG. 15. The neutron and electron chemical potentials in beta stable matter according to models A18+ δv + UIX* (full line) and A18+ δv (dashed line). Threshold densities for the appearance of noninteracting hyperons are marked by horizontal line segments.

$$
\begin{aligned}
\mu_{\Sigma^-} &= \mu_{\Delta^-} = \mu_n + \mu_e \\
\mu_\Lambda &= \mu_{\Sigma^0} = \mu_{\Delta^0} = \mu_n \\
\mu_{\Sigma^+} &= \mu_{\Delta^+} = \mu_p = \mu_n - \mu_e \\
\mu_{\Delta^{++}} &= \mu_n - 2\mu_e
\end{aligned}
$$



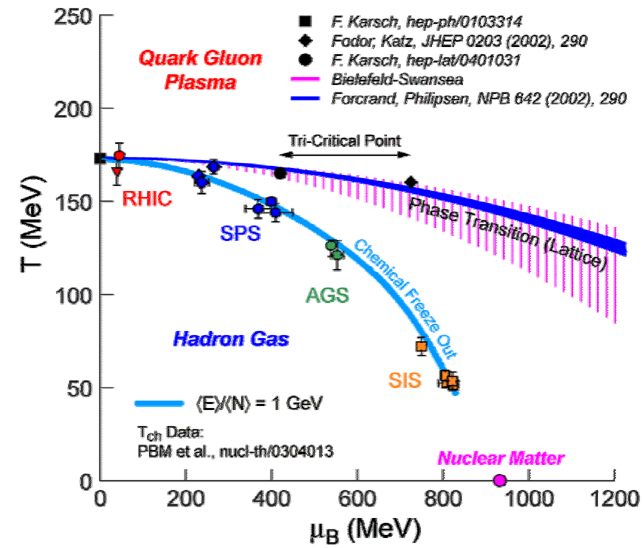
Interactions included

Balberg, Licthenstadt, & Cook Ap.J. Suppl., 121, p515

Quark matter

- matter compressed beyond 5 to 10ρ_0
- deconfining transition: quark matter
  - T=0 (distinct from RHIC physics where T¼175 MeV)
  - QCD is asymptotically free

    $\alpha_s(Q^2)$»$1/\log(Q^2/\Lambda^2)$
  - weakly interacting quarks
- nature of phase transition depends on interactions
- experimental observations
  - RHIC, SPS, AGS, SIS
- lattice calculations
  - at high T, low μ_B
  - difficult at low T, large μ_B due to `sign problem' in Monte Carlo simulation of fermions
- Our perspective
  - quark matter will be relevant at some ρ
  - what observational consequences for phenomenological models?



C. Gagliardi, QNP2004

Unpaired quark matter

- free energy (– pressure)
  - ignore charm, bottom, top quarks

$$\Omega_{UQM}(\mu,\mu_e) \;=\; \frac{3}{\pi^2}\int_0^{\nu_u} dp\, p^2 (p-\mu_u) + \frac{3}{\pi^2}\int_0^{\nu_d} dp\, p^2 (p-\mu_d) + \frac{3}{\pi^2}\int_0^{\nu_s} dp\, p^2 (\sqrt{p^2+m_s^2}-\mu_s)$$

massless

up+down quarks

massive

strange quarks

$$\begin{aligned}
\mu &= \mu_n/3 \\
\nu_u^2 &= \mu_u^2 - m_u^2, \quad \mu_u = \mu - \frac{2}{3}\mu_e \\
\nu_d^2 &= \mu_d^2 - m_d^2, \quad \mu_d = \mu + \frac{1}{3}\mu_e \\
\nu_s^2 &= \mu_s^2 - m_s^2, \quad \mu_s = \mu - \frac{1}{3}\mu_e
\end{aligned}$$

- bag pressure -- exclusion of non-perturbative QCD costs energy

$$\begin{aligned}
\Omega_{UQM}(\mu,\mu_e) &\;\rightarrow\; \Omega_{UQM}(\mu,\mu_e) + B \\
B &\;\approx\; 200\,\mathrm{MeV/fm^3}
\end{aligned}$$

- ground state – three independent Fermi spheres

Fermi surface and BCS pairing

- absence of interactions – Fermi surface is stable
  - $F=E-\mu N$, $E_F=\mu$) $\Delta F=0$, $\Delta N=1$
- introduce arbitrarily weak attractive interaction in any channel
  - no free energy cost to make pairs
  - modes near Fermi surface will *pair*
  - pairs are bosons) condensate
  - ground state = superposition of states with all numbers of pairs
  - Fermion number symmetry is broken
- QCD

e.g.

$$\mathbf{3} \sim q = \begin{pmatrix} u \\ d \\ s \end{pmatrix} \qquad u = u_{\alpha,c}, \quad \alpha = 1,\ldots,4 \quad c = R, G, B$$

$$\bar{\mathbf{3}} \oplus \mathbf{6} = \mathbf{3} \otimes \mathbf{3}$$

$$\bar{R} = \frac{1}{\sqrt{2}}(BG - GB) \qquad RR, \ldots$$

$$\bar{G} = \frac{1}{\sqrt{2}}(BR - RB)$$

$$\bar{B} = \frac{1}{\sqrt{2}}(RG - GR)$$

- perturbative gluon interaction

$$T\cdot T = \sum_c T^c T^c \qquad \langle\bar{\mathbf{3}}|T\cdot T|\bar{\mathbf{3}}\rangle = -\frac{2}{3} \rightarrow \text{color superconductivity}$$

$$T^c = \frac{\lambda^c}{2} \qquad \langle 6|T\cdot T|6\rangle = +\frac{1}{3}$$

The gap equation

$$\Sigma(k) = -\frac{1}{(2\pi)^4}\int d^4q\, M^{-1}(q) D(k-q),$$

Ansatz:

$$M(q) = M_{\text{free}} + \Sigma = \begin{pmatrix} \not{q} + \mu\gamma_0 & \gamma_0\Delta\gamma_0 \\ \Delta & (\not{q} - \mu\gamma_0)^T \end{pmatrix}$$

$$\Psi = \begin{pmatrix} q \\ \bar{q}^T \end{pmatrix}, \qquad \langle q C\gamma_5 q\rangle$$

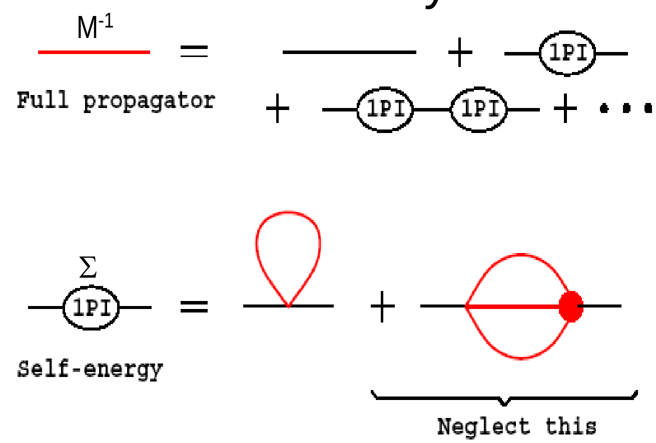
$$1 = K\int_0^\Lambda k^2 dk \frac{1}{\sqrt{(k-\mu)^2 + \Delta^2}}$$

$$\Delta \sim \Lambda \exp\left(\frac{\text{const}}{K\mu^2}\right)$$

non-analytic behavior
in the coupling, K
appears at no finite order
in perturbation theory

$$\Delta \sim 10 \text{ to } 100 \text{ MeV}$$

Mean Field Theory:



Mark Alford

Interactions:

- pQCD at μ»10⁶ MeV
- instanton vertex
  - -four fermion int.
- Nambu—Jona-Lisinio
  - -QCD w/out gluon propagator

Color-Flavor-Locked (CFL) phase

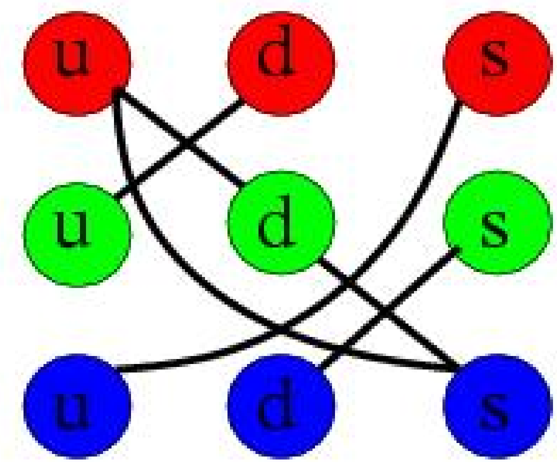
Alford, Rajagopal, Wilczek, hep-ph/9804403

- choose a particular pairing arrangement for 3 massless quarks

$$\Delta_{ij}^{ab} = \langle q_i^a q_j^b \rangle$$
$$\propto C\gamma_5 \left[\epsilon^{abX} \epsilon_{ijX} + \kappa(\delta_i^a \delta_j^b + \delta_j^a \delta_i^b) \right]$$

color and flavor locked in $(\overline{\mathbf{3}}_A, \overline{\mathbf{3}}_A)$

$(\mathbf{6}_S, \mathbf{6}_S)$



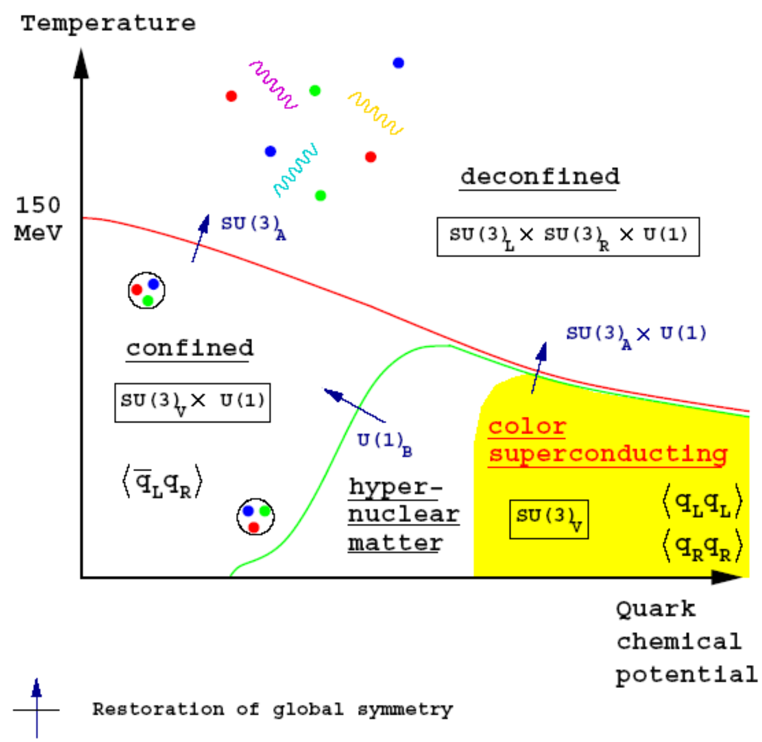
- breaks symmetry of chiral QCD Lagrangian

$$SU(3)_{color} \otimes SU(3)_L \otimes SU(3)_R \otimes U(1)_B$$
$$\downarrow$$
$$SU(3)_{color+L+R} \otimes Z_2$$

Symmetry breaking in CFL phase

- color is completely broken) all 8 gluons become massive
- all quark modes are gapped
  - nine quasi-quarks **8©1** corresponding to unbroken SU(3)$_{\mathrm{color+L+R}}$
  - two gaps – singlet > octet
- electromagnetism U(1)$_Q$ mixes with flavor symmetry

) $\tilde{Q} = Q + \frac{1}{\sqrt{3}} T_8$ is conserved
- Broken global symmetries
  - chiral symmetry
    - pseudoscalar octet of chiral Goldstone bosons, “K,π,η”
  - baryon number
    - hudsudsi $\neq$ 0) CFL superfluid



CFL properties

- consider $m_u = m_d = 0$, m_s ¿ μ
- Fermi momenta of u,d,s are the same
  - minimizes free energy for small m_s
  - maximizes overlap for pairing
- ground state is charge neutral
  - equal fermi momenta) equal numbers of u,d,s
  - no leptons required
- common Fermi momentum

$$
\begin{aligned}
3\mu &= 2\nu + \sqrt{\nu^2 + m_s^2} \\
=> \nu &= 2\mu - \sqrt{\mu^2 + \frac{m_s^2}{3}}
\end{aligned}
$$

CFL EoS

- free energy

$$\Omega_{CFL}(\mu,\mu_e) \;=\; \Omega_{CFL}^{quarks}(\mu) + \Omega_{CFL}^{GB}(\mu,\mu_e) + \Omega_{CFL}^{leptons}(\mu_e)$$

- Quarks – kinetic + gap + bag constant

$$\begin{aligned}\Omega_{CFL}^{quarks}(\mu) \;&=\; \frac{6}{\pi^2}\int_0^{\nu} dp\, p^2 (p-\mu) && \text{up+down massless}\\ &+\; \frac{3}{\pi^2}\int_0^{\nu} dp\, p^2 \left(\sqrt{p^2+m_s^2}-\mu\right) && \text{strange}\\ &-\; \frac{3\Delta^2\mu^2}{\pi^2} + B && \text{gap + bag constant}\end{aligned}$$

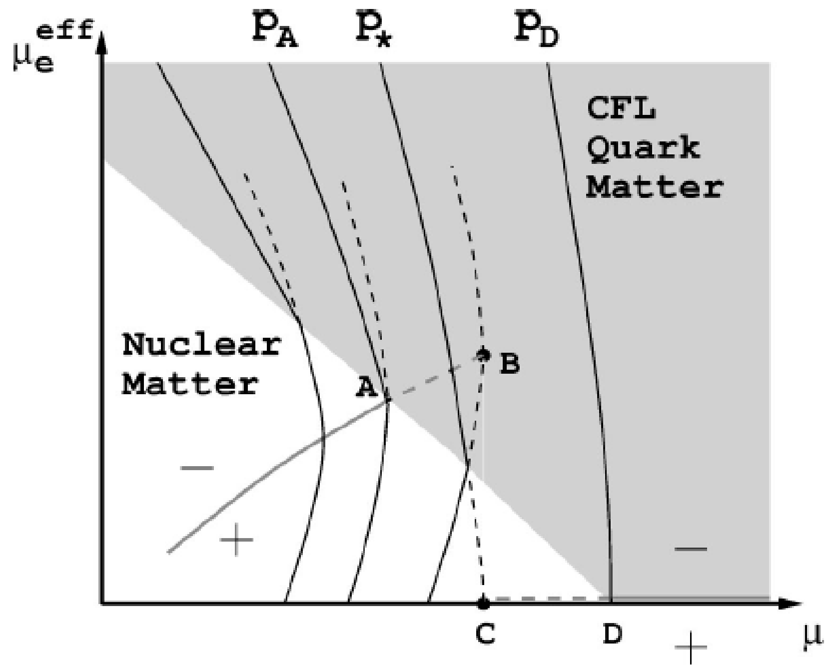
- Goldstone bosons – condensate non-zero from broken symm.

$$\begin{aligned}\Omega_{CFL}^{GB}(\mu,\mu_e) \;&=\; -\frac{1}{2} f_\pi^2(\mu)\mu_e^2 \left(1-\frac{m_\pi^2}{\mu_e^2}\right)^2\\ f_\pi^2(\mu) \;&=\; c\mu^2\\ m_\pi^2 \;&=\; \frac{3\Delta^2}{\pi^2 f_\pi^2} m_s(m_u+m_d)\end{aligned}$$

CFL – nuclear interface

- single sharp interface – point B
  - beta stable charge neutral nuclear matter interface with chargeless CFL
  - charged boundary layers

$$\Omega_{nuclear}(\mu(B), \mu_e) = \Omega_{CFL}(\mu(B))$$

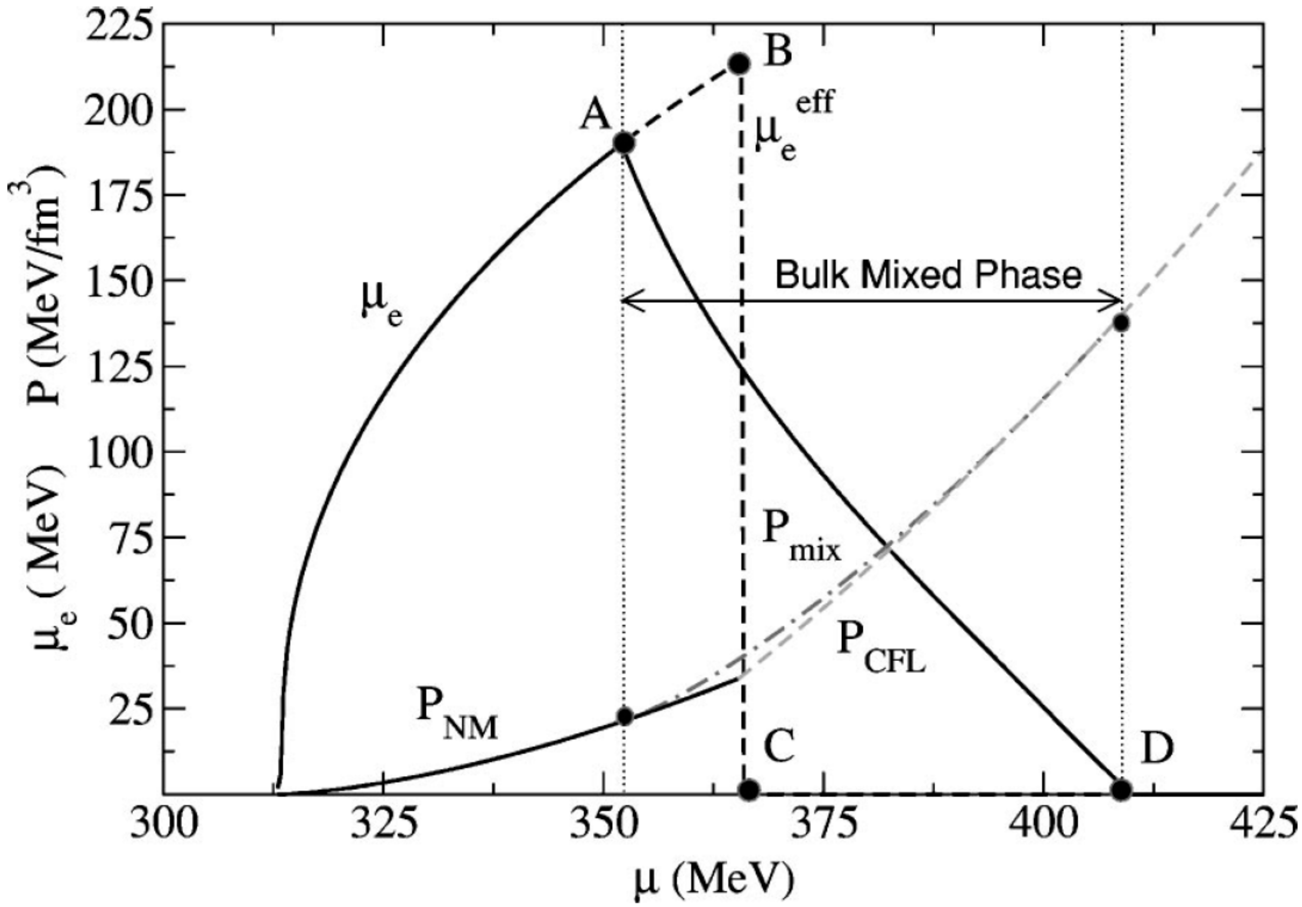


- mixed phase region
  - neither NM phase nor CFL phase is charge-neutral
  - bulk is charge-neutral
  - Coulomb and surface energies could make the mixed phase unfavorable

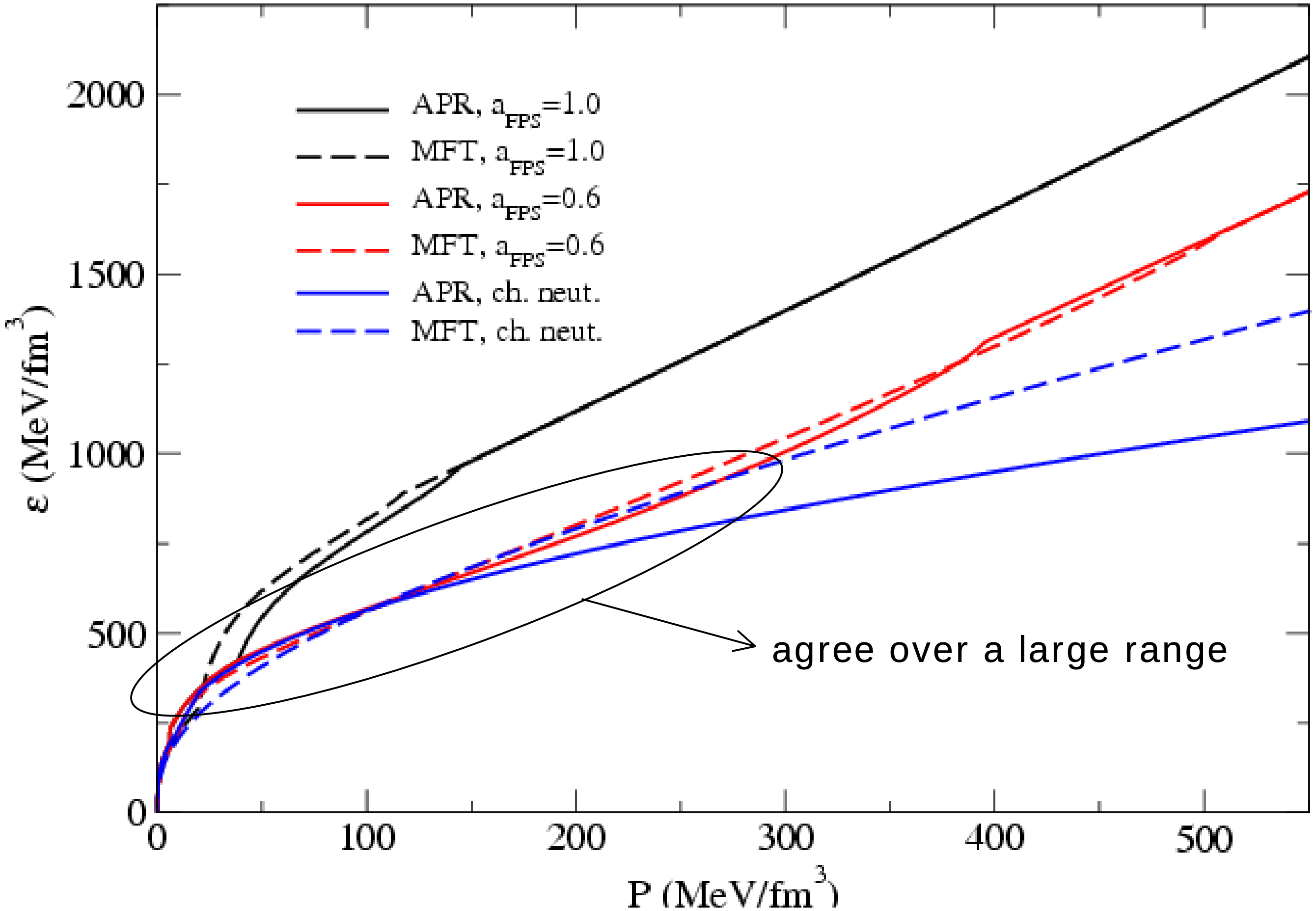
$$\Omega_{nuclear}(\mu, \mu_e) = \Omega_{CFL+Kaons}(\mu, \mu_e)$$

- Assume mixed phase region obtains: make a star

Mixed phase properties



EoS – Mixed phase



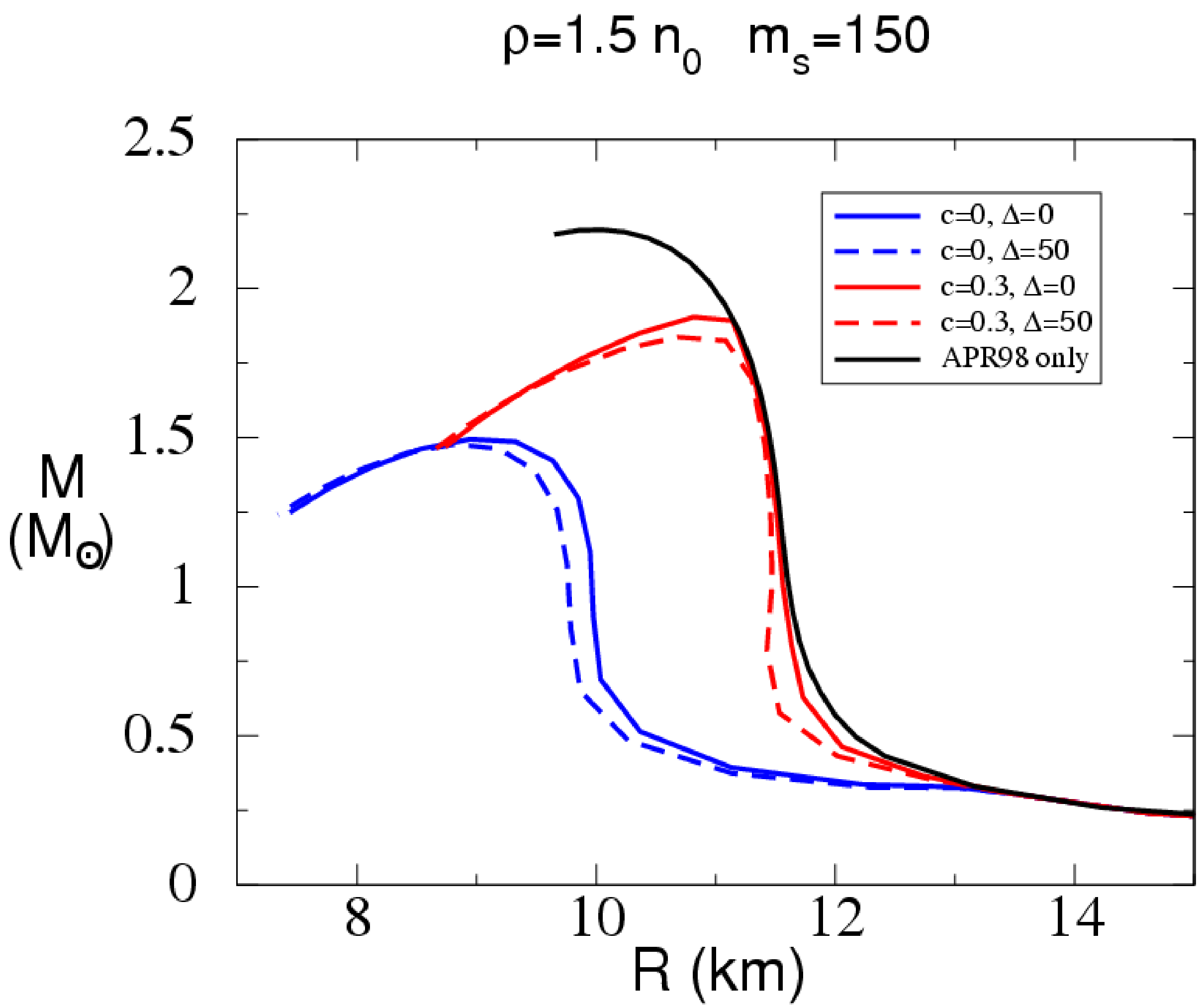
perturbative corr.

$$\Omega_{CFL}^{quarks} \rightarrow a_{FPS}\Omega_{CFL}^{quarks}$$

$$a_{FPS} = 1 - 2\frac{\alpha_s}{\pi}$$

Masquerading hybrid stars

- perturb. corr. stiffen EoS
- M(R) for pure NM closely follows hybrid curve
- Require extremely precise Radius determination
- transport properties more sensitive to exotic phases
  - mean free paths
  - conductivities
  - viscosities
  - opacities



Things I didn't cover

- 2SC
  - only light quarks pair
- gapless superconductivity
- color flavor unlocking and crystalline color superconductivity
  - strange quark mass effect
  - "LOFF" phase
  - Cooper pairs have non-zero total momentum
  - spatially periodically varying gap
- gamma ray bursters
- glitches

- Many more...